7. Seepage through soils

7.1 Introduction

'Seepage' is defined as the flow of a fluid, usually water, through a soil under a hydraulic gradient. A hydraulic gradient is supposed to exist between two points if there exists a difference in the 'hydraulic head' at the two points. By hydraulic head is meant the sum of the position or datum head and pressure head of water. The discussion on flow nets and seepage relates to the practical aspect of controlling groundwater during and after construction of foundations below the groundwater table, earth dam and weirs on permeable foundations.

The interaction between soils and percolating water has an important influence on:
1. The design of foundations and earth slopes,
2. The quantity of water that will be lost by percolation through a dam or its subsoil.

Foundation failures due to 'piping' are quite common. Piping is a phenomenon by which the soil on the downstream sides of some hydraulic structures get lifted up due to excess pressure of water. The pressure that is exerted on the soil due to the seepage of water is called the seepage force or pressure. In the stability of slopes, the seepage force is a very important factor. Shear strengths of soils are reduced due to the development of neutral stress or pore pressures. A detailed understanding of the hydraulic conditions is therefore essential for a satisfactory design of structures.

7.2 Two dimensional flow- Laplace equation

In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In such cases, the groundwater flow is generally calculated by the use of graphs referred to as flow nets. The concept of the flow net is based on Laplace’s equation of continuity, which governs the steady flow condition for a given point in the soil mass.

Laplace’s Equation of Continuity

To derive the Laplace differential equation of continuity, let us consider a single row of sheet piles that have been driven into a permeable soil layer, as shown in Figure a. The row of sheet piles is assumed to be impervious. The steady state flow of water from the upstream to the downstream side through the permeable layer is a two-dimensional flow.

For flow at a point , we consider an elemental soil block. The block has dimensions , and (length is perpendicular to the plane of the paper); it is shown in an enlarged scale in Figure b. Let and be the components of the discharge velocity in the horizontal and vertical directions, respectively.

The rate of flow of water into the elemental block in the horizontal direction is equal to . and in the vertical direction it is . The rates of outflow from the block in the horizontal and vertical directions are, respectively,
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Assuming that water is incompressible and that no volume change in the soil mass occurs, we know that the total rate of inflow should equal the total rate of outflow. Thus,

\[ \left( v_x + \frac{\partial v_x}{\partial x} \right) dz
dy + \left( v_z + \frac{\partial v_z}{\partial z} \right) dx
dy = 0 \]

or

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \]

With Darcy’s law, the discharge velocities can be expressed as

\[ v_x = k_x \frac{\partial h}{\partial x} \]
\[ v_z = k_z \frac{\partial h}{\partial z} \]

where \( k_x \) and \( k_z \) are the hydraulic conductivities in the horizontal and vertical directions, respectively.

Thus, we can write

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \]

If the soil is isotropic with respect to the hydraulic conductivity—that is, \( k_x = k_z \)—the preceding continuity equation for two-dimensional flow simplifies to

\[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \]

The above equation is the Laplace Equation for homogenous soil. It says that the change of gradient in the x-direction plus the change of gradient in the z-direction is zero. The solution of this equation gives a family of curves meeting at right angles to each other. One family of these curves represents flow lines and the other equipotential lines.

For reference only (Make your own notes)
7.3 Flow nets

A flow net for an isometric medium is a network of flow lines and equipotential lines intersecting at right angles to each other. The path which a particle of water follows in its course of seepage through a saturated soil mass is called a flow line.

Equipotential lines are lines that intersect the flow lines at right angles. At all points along an equipotential line, the water would rise in piezometric tubes to the same elevation known as the piezometric head.

A combination of a number of flow lines and equipotential lines is called a flow net.

The properties of a flow net can be expressed as given below:
1. Flow and equipotential lines are smooth curves.
2. Flow lines and equipotential lines meet at right angles to each other.
3. No two flow lines cross each other.
4. No two flow or equipotential lines start from the same point.

There are many methods that are in use for the construction of flow nets. Some of the important methods are
1. Analytical method,
2. Electrical analog method,
3. Scaled model method,

The analytical method, based on the Laplace equation although rigorously precise, is not universally applicable in all cases because of the complexity of the problem involved. The mathematics involved even in some elementary cases is beyond the comprehension of many design engineers. Although this approach is sometimes useful in the checking of other methods, it is largely of academic interest.

The electrical analogy method has been extensively made use of in many important design problems. However, in most of the cases in the field of soil mechanics where the estimation of seepage flows and pressures are generally required, a more simple method such as the graphical method is preferred.

Scaled models are very useful to solve seepage flow problems. Soil models can be constructed to depict flow of water below concrete dams or through earth dams. These models are very useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models.

The graphical method developed by Forchheimer (1930) has been found to be very useful in solving complicated flow problems. A. Casagrande (1937) improved this method by incorporating many suggestions. The main drawback of this method is that a good deal of practice and aptitude are essential to produce a satisfactory flow net. In spite of these drawbacks, the graphical method is quite popular among engineers.

Flow Net Construction

Flow nets are constructed in such a way as to keep the ratio of the sides of each block bounded by two flow lines and two equipotential lines a constant. If all the sides of one such block are
equal, then the flow net must consist of squares. The square block referred to here does not constitute a square according to the strict meaning of the word; it only means that the average width of the square blocks are equal.

The area bounded by any two neighboring flow lines is called a flow channel. If the flow net is constructed in such a way that the ratio of width of square blocks \((a/b)\) remains the same for all blocks, then it can be shown that there is the same quantity of seepage in each flow channel.

**Guidelines for drawing flow nets**

- Draw to a convenient scale the cross sections of the structure, water elevations, and soil deposit profiles.
- Establish boundary conditions that is, Identify impermeable and permeable boundaries. The soil and impermeable boundary interfaces are flow lines. The soil and permeable boundary interfaces are equipotential lines.
- Draw one or two flow lines and equipotential lines near the boundaries. Sketch intermediate flow lines and equipotential lines by smooth curves adhering to right-angle intersections such that area between a pair of flow lines and a pair of equipotential lines is approximately a curvilinear square grid.
- Where flow direction is a straight line, flow lines are equal distance apart and parallel. Also, the flownet in confined areas between parallel boundaries usually consists of flow lines and equipotential lines that are elliptical in shape and symmetrical.
- Try to avoid making sharp transition between straight and curved sections of flow and equipotential lines. Transitions must be gradual and smooth. Continue sketching until a problem develops.
- Successive trials will result in a reasonably consistent flow net. In most cases, 3 to 8 flow lines are usually sufficient. Depending on the number of flow lines selected, the number of equipotential lines will automatically be fixed by geometry and grid layout.
7.4 Unconfined flow

Determination of seepage discharge from a flow net

If $H$ is the net hydraulic head of flow, the quantity of seepage due to flow may be estimated by drawing flow net part of which is shown in Figure. With reference to Figure following terms may be defined in order to estimate quantity of seepage.

![Flow net and its characteristics](image)

$N_d$ = Number of equipotential drops, that is, number of squares between two adjacent
$N_f$ = Number of flow channels that is, number of squares between two adjacent equipotential
   lines from one boundary streamline to the other boundary streamline

$\Delta q$ = flow through one flow channel (between two adjacent streamlines)

$\Delta h$ = head loss between two adjacent equipotential lines

Consider a flow grid of dimension $a \times b$ (circled), where

- $b = \Delta l$ = distance between equipotential lines
- $a = A$ = area across flow channel

Head loss for every potential drop: $\Delta h = \frac{H}{N_d}$ and $i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b}$

So, $i = \frac{H}{bN_d}$

Flow per channel as per Darcy’s law,

$$\Delta q = vi = k \Delta A = k \frac{H}{bN_d} a$$

Total flow per unit width across each flow channel is

$$q = kH \left( \frac{N_f}{N_d} \right) \left( \frac{a}{b} \right)$$

If $a=b$, then

$$q = kH \left( \frac{N_f}{N_d} \right)$$
Determination of Hydrostatic Pressure

Hydrostatic pressure in the flow area can be analyzed by means of flow net. In the below figure the equipotential lines are numbered in increasing order from downstream to upstream from 0 for the downstream line (taken as datum). The head loss between 2 adjacent equipotential lines is $\Delta h$, while the total head loss is $H$.

Since, $\Delta h$ is constant for every two adjacent equipotential lines, then,

$$\Delta h = \frac{H}{N_d}$$

Using the numbers assigned to the equipotential lines, the total head for each equipotential line can be calculated as:

$$h = \frac{n_d}{N_d} H$$

where,

$n_d$ = the number of the corresponding equipotential line

In the figure, at point A, $n_d=1$ and $N_d=3$, the total head at A will be, $h_A = \frac{1}{3} H$

The pressure head at A: $h_{pA} = h_A - (-h_e A)$

The hydrostatic pressure at A is: $u_A = h_{pA} \gamma_w$

Determination of uplift pressure

The uplift pressure at point $a$ in the figure may be expressed as

$$u_a = h_a \gamma_w = (h_t + D - \Delta h) \gamma_w$$

Similarly, the uplift pressure at any other point say $e$ may be estimated from the expression, $u_e = (h_t + D - n_d \Delta h) \gamma_w$

For reference only (Make your own notes)
7.5 Seepage in anisotropic soil condition

Laplace’s equation for flow through soil was derived under the assumption that permeability is the same in all directions. Before stipulating this condition in the derivation, the equation was:

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \]

This may be reduced to the form:

\[ \frac{\partial^2 h}{\partial z^2} + \left( \frac{k_z}{k_x} \right) \frac{\partial^2 h}{\partial x^2} = 0 \]

By changing the co-ordinate \( x \) to \( x_T \) such that \( x_T = \sqrt{\frac{k_z}{k_x}} \cdot x \); We get

\[ \frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x_T^2} = 0 \]

which is once again the Laplace’s equation in \( x_T \) and \( z \).

In other words, the profile is to be transformed according to the relationship between \( x \) and \( x_T \) and the flow net sketched on the transformed section.

From the transformed section, the rate of seepage can be determined using with exception that \( k_e \) is to be substituted for \( k \)

![Flow in anisotropic soil](image)

Discharge through Natural Section:

\[ q_N = k_x \cdot i_N \cdot A = k_x \cdot \frac{\Delta h}{l} \cdot b \]

Discharge through Transformed Section:

\[ q_T = k_e \cdot i_T \cdot A = k_e \cdot \frac{\Delta h}{l \sqrt{k_z/k_x}} \cdot b = \Delta h \cdot k_e \]

In the transformed section the rectangles are of equal sides, hence:

\[ b = l \cdot \sqrt{k_z/k_x} \]

Substituting the value of \( b \) in equation for \( q_N \),

\[ q_N = k_x \cdot \sqrt{k_z/k_x} \cdot \Delta h \]

Since \( q_T = q_N \), therefore,

\[ k_e = \sqrt{k_x \cdot k_z} \]
7.6 Seepage through an earth dam on an impervious base

In almost all problems concerning seepage beneath a sheet pile wall or through the foundation of a concrete dam all boundary conditions are known. However, in the case of seepage through an earth dam the upper boundary or the uppermost flow line is not known. This upper boundary is a free water surface and will be referred to as the line of seepage or phreatic line.

The seepage line may therefore be defined as the line above which there is no hydrostatic pressure and below which there is hydrostatic pressure. Therefore phreatic line is the top flow line which separates saturated and unsaturated zones within the body of the earth dam.

In the design of all earth dams, the following factors are very important.

1. The seepage line should not cut the downstream slope.
2. The seepage loss through the dam should be the minimum possible.

The two important problems that are required to be studied in the design of earth dams are:

1. The prediction of the position of the line of seepage in the cross-section.
2. The computation of the seepage loss.

If the line of seepage is allowed to intersect the downstream face much above the toe, more or less serious sloughing may take place and ultimate failure may result. This mishap can be prevented by providing suitable drainage arrangements on the downstream side of the dam.

Therefore the problem of computation of the seepage loss through an earth dam primarily involves prediction of the position of the line of seepage in the cross-section.

Locating Phreatic Line

It has been noticed from experiments on homogeneous earth dam models that the line of seepage assumes more or less the shape of a parabola. Also, assuming that hydraulic gradient \( i \) is equal to the slope of the free surface and is constant with depth (Dupit's theory), the resulting solution of the phreatic surface is parabola.

In some sections a little divergence from a regular parabola is required particularly at the surfaces of entry and discharge of the line of seepage. The properties of the regular parabola which are essential to obtain phreatic line are depicted in Figure aside.

For reference only (Make your own notes)
Every point on the parabola is equidistance from focus and directrix, Therefore,

\[ FA = AB \]

Also,

\[ FG = GE = p = \frac{s}{2} \]

Focus = (0,0)

Any point \( A(x, z) \) on the parabola is given by,

\[ FA = \sqrt{(x - 0)^2 + (z - 0)^2} = AB \]

\[ AB = (2p + x) \]

\[ x^2 + z^2 = (2p + x)^2 \]

Thus,

\[ x = \frac{z^2 - 4p^2}{4p} \]

**Phreatic line for an earth dam without toe filter**

In the case of a homogeneous earth dam resting on an impervious foundation with no drainage filter, the top flow line ends at some point on the downstream face of the dam; the focus of the base parabola in this case happens to be the downstream toe of the dam itself as shown in Figure below:

![Figure: Phreatic line for an earth dam without toe filter](image)

The following are the steps in the graphical determination of the top flow line for a homogeneous dam resting on an impervious foundation without filters:

1. Draw the earth dam section and upstream water level \( H \) to some convenient scale. Let Point-2 is the point on the upstream face coinciding with water level.
2. Let $\Delta$ be the horizontal distance between point-2 and upstream heel of the dam. Locate Point-1 at a distance of 0.3 times $\Delta$ from Point-2 on the water surface. That is distance 1-2 is 0.3$\Delta$.

3. Focus of the base parabola is located at the downstream toe of the dam, that is Point-0 (distance 0 - 1 is d). Select x-z reference axis with focus 0 as origin.

4. Directrix of the parabola is at distance $2p$ from the focus 0, where $p$ is given by, considering from point 1,
   \[ p = \frac{1}{2}(\sqrt{d^2 + H^2} - d) \]

5. By choosing suitable values of $z$-ordinates (for example; $0.2H$, $0.4H$, ... & $H$) compute the $x$-ordinates of the base parabola using the relation,
   \[ x = \frac{z^2 - 4p^2}{4p} \]

6. Join all these points to get base parabola starting from Point-1 and concluding at a point midway between focus-0 and directrix. This parabola will be correct for the central portion of the top flow line. Necessary corrections at the entry on the upstream side and at exist on the downstream side are to be made.

7. Upstream correction: The portion of the top flow line at entry is sketched visually to meet the boundary condition there that is phreatic line meets perpendicularly with the upstream face, which is a boundary equipotential and the phreatic line is made to meet the base parabola tangentially at a convenient point.

8. Downstream correction (Casagrande's method): The breakout point on the downstream discharge face may be determined by measuring out $L$ from the toe along the face. If $\beta$ is the downstream slope angle then $L$ may be computed from the following equations,

9. Finally the quantity of seepage flow through may be compute from the following equations,
   \[ \text{For } \beta < 30^\circ, \quad q = kL \sin \beta \tan \beta \]
   \[ \text{For } 30^\circ < \beta < 90^\circ, \quad q = kL \sin^2 \beta \]

   Where, $k$ is the coefficient of permeability of the dam material.

   \[ \text{For } \beta < 30^\circ, \quad L = \frac{d}{\cos \beta} - \frac{\sqrt{d^2 + H^2}}{\cos^2 \beta \sin \beta} \]
   \[ \text{For } 30^\circ < \beta < 90^\circ, \quad L = \sqrt{H^2 + d^2} - \sqrt{d^2 + H^2 \cot^2 \beta} \]
Phreatic line for an earth dam with toe filter

The following are the steps in the graphical determination of the top flow line for a homogeneous dam with a toe filter: [Refer to Figures (a), (b), (c), (d), & (e)]

1. Draw the earth dam section and upstream water level (H) to some convenient scale.
2. Locate Point-B, the point on the upstream slope coinciding with water level.
3. Let A be the horizontal distance between point B and upstream heel of the dam. Locate Point-A at a distance of 0.3 times A from Point-B on the water surface. That is distance AB is 0.3A [Refer to Figure (a)]
4. Select F as the focus of the parabolic phreatic line, Point-F is located at the intersection of the bottom flow line and the downstream toe filter. Let horizontal distance between points A & F be d i.e., AF = d

For reference only (Make your own notes)
5. Locate Point-\(G\) on the directrix of the parabola, located a distance \(2p\) from the focal point, Point-\(F\), that is \(FG = 2p\) where, [Refer to Figure (b)]
\[
p = \frac{1}{2}(\sqrt{d^2 + H^2} - d)
\]

6. Select base of the dam and directrix as \(X\) & \(Z\) axes

7. By choosing suitable values of \(z\)-ordinates (for example; \(0.2H, 0.4H \ldots \& H\)) compute the \(x\)-ordinates of the base parabola using the relation,
\[
x = \frac{z^2 - 4p^2}{4p}
\]

Figure: Phreatic line for an earth dam with toe filter

For reference only (Make your own notes)
Thus \( z_1, z_2, z_3, z_4 \ldots \) are computed for the ordinates \( x_1, x_2, x_3, x_4 \ldots \) respectively [Refer to Figure (c)].

8. Join all such located points to get basic parabola. This parabola meets toe filter (equipotential line) orthogonally at midpoint of \( FG \) that is at a distance \( p \) from \( F \) (vertex \( K \) of the parabola). Joint points \( K-0-1-2-3-4-A \) to get parabola \( ABK \) [Refer to Figure (d)].

9. Apply modification to phreatic line at the entry Point-B on the upstream slope which is an equipotential line. Draw line perpendicular to upstream slope starting from \( B \) and meets the base parabola smoothly and tangentially at a convenient point say, \( C \). Complete the phreatic line \( BCK \) (top flow line) by joining \( BC \) erase remaining portion of the base parabola [Refer to Figure (e)].

10. Finally the quantity of seepage flow through may be compute from the following equations. Let the distance between \( F \) & \( G \) is \( S \) i.e., the distance between focus and directrix, \( \therefore S = 2 \times p \)
Then the quantity of seepage through unit length of dam is,
\[
q = kS = k \times (2 \times p) = 2kp
\]
Where, \( k \) is the coefficient of permeability of the dam material.

### 7.7 Flow through non homogeneous sections

In case of flow perpendicular to soil strata, the loss of head and rate of flow are influenced primarily by the less pervious soil whereas in the case of flow parallel to the strata, the rate of flow is essential controlled by comparatively more pervious soil.

Figure given shows a flow channel and part of a flow net, from soil \( A \) to soil \( B \). The permeability of soil \( A \) is greater than that of soil \( B \). By the principle of continuity, the same rate of flow exists in the flow channel in soil \( A \) as in soil \( B \).

By means of this, relationship between the angles of incidence of the flow paths with the boundary of the two flow channels can be determined. Not only does the direction of flow change at the boundary between soils with different permeabilities, but also the geometry of the figures in the flow net changes. As can be seen from Fig., the figures in soil \( B \) are not squares as in soil \( A \), but are rectangles.
7.8 Prevention of erosion- protective filters

Filter drains are required on the downstream sides of hydraulic structures and around drainage pipes. A properly graded filter prevents the erosion of soil in contact with it due to seepage forces. To prevent the movement of erodible soils into or through filters, the pore spaces between the filter particles should be small enough to hold some of the protected materials in place.

Taylor (1948) shows that if three perfect spheres have diameters greater than 6.5 times the diameter of a small sphere, the small spheres can move through the larger as shown in Fig. (a). Soils and aggregates are always composed of ranges of particle sizes, and if pore spaces in filters are small enough to hold the 85 per cent size (D85) of the protected soil in place, the finer particles will also be held in place as exhibited schematically in Fig. (b).

The requirements of a filter to keep the protected soil particles from invading the filter significantly are based on particle size. These requirements were developed from tests by Terzaghi which were later extended by the U.S. Army Corps of Engineers (1953). The resulting filter specifications relate the grading of the protective filter to that of the soil being protected by the following:

\[
\frac{D_{15 \text{ filter}}}{D_{85 \text{ soil}}} \leq 4, \quad 4 < \frac{D_{15 \text{ filter}}}{D_{15 \text{ soil}}} \leq 20, \quad \frac{D_{50 \text{ filter}}}{D_{50 \text{ soil}}} \leq 25
\]

The criteria may be explained as follows:

1. The 15 per cent size \((D_{15})\) of filter material must be less than 4 times the 85 per cent size \((D_{85})\) of a protected soil. The ratio of \(D_{15}\) of a filter to \(D_{85}\) of a soil is called the piping ratio.

2. The 15 per cent size \((D_{15})\) of a filter material should be at least 4 times the 15 per cent size \((D_{15})\) of a protected soil but not more than 20 times of the latter.

3. The 50 per cent size \((D_{50})\) of filter material should be less than 25 times the 50 per cent size \((D_{50})\) of protected soil.

A typical grain size distribution curve of a protected soil and the limiting sizes of filter materials for constructing a graded filter is given in Fig. below. The size of filter materials must fall within the two curves \(C_2\) and \(C_3\) to satisfy the requirements.
Figure: Grain size distribution curves for graded filter and protected materials.