Ch-3. Lateral Earth Pressure Theories and Retaining Walls

3.1. Introduction

A soil mass is stable when the slope of the surface of the soil mass is flatter than the safe slope. At some locations, due to limitation of space, it's not possible to provide flat slope & the soil is to be retained at a slope steeper than the safe one. In such cases, a retaining structure is required to provide lateral support to the soil mass.

A retaining structure is a permanent or temporary structure which is used for providing lateral support to the soil mass or other materials. Some of the examples of retaining structures used in soil & foundation engineering are: Retaining wall, Sheet piles, Anchored Bulkheads, Sheeting & Basement wall, etc. In the absence of a retaining structure, the soil on the higher side would have a tendency to slide and may not remain stable.

The design of the retaining structure requires the determination of the magnitude & line of action of the lateral earth pressure. The magnitude of the lateral earth pressure depends upon a number of factors, such as the mode of movement of the wall, the flexibility of the wall, the properties of the soil, the drainage conditions. For convenience, the retaining wall is assumed to be rigid & the soil structure interaction effect is neglected which arises due to the flexibility of the wall.

The pressure or force exerted by soil on any boundary is called the earth pressure. When the earth pressure acts on the side (back or face) of a retaining wall, it is known as the Lateral earth pressure. The magnitude of the lateral earth pressure depends upon the movement of the retaining wall relative to the backfill & upon the nature of the soil.

Figure- Gravity and Cantilever retaining wall under construction

3.2 Effect of wall movement on Earth Pressure

The mass is bounded by a frictionless wall of height AB. A soil element located at a depth z is subjected to a vertical effective pressure, and a horizontal effective pressure. There are no shear stresses on the vertical and horizontal planes of the soil element. Let us define the ratio of to as a nondimensional quantity $K$, or

$$K = \frac{\sigma_h}{\sigma_v}$$

For reference Only (Make your own notes)
Case I - Coefficient of earth pressure at rest ($K_a$):
When the wall is rigid and unyielding, the soil mass is in a state of rest and there are no deformations and displacements. The earth pressure corresponding to this state is called the earth pressure at rest.

Case II- Active earth pressure coefficient ($K_a$):
If the wall rotates about its toe, thus moving away from the backfill, the soil mass expands, resulting in a decrease of the earth pressure. When the wall moves away from the backfill, a portion of the backfill located next to the retaining wall tends to break away from the rest of the soil mass and tends to move downwards and outwards relative to the wall. Since the shearing resistance is mobilized in directions away from the wall, there is a resultant decrease in earth pressure which continues until at a certain amount of displacement, failure will occur in the backfill and slip surfaces will be developed. At this stage the entire shearing resistance has been mobilized. The force acting on the wall at this stage does not decrease any more beyond this point even with further wall movement. This force is called the active earth pressure.

Case III- Passive earth pressure coefficient ($K_p$):
On the other hand, if the wall is pushed towards the backfill, the soil is compressed and the soil offers resistance to this movement by virtue of its shearing resistance. Since the shearing resistance builds up in directions towards the wall, the earth pressure gradually increases. If this force reaches a value which the backfill cannot withstand, failure again ensues and slip surface develop. The pressure reaches a maximum value when the entire shearing resistance has been mobilized and does not increase any more with further wall movement. This pressure is called the passive earth pressure.

Fig. Variation of Magnitude of Lateral Earth Pressure with Wall Tilt

For reference Only (Make your own notes)
3.3 Earth Pressure at Rest

The value of $K_o$ depends upon the relative density of the sand and the process by which the deposit was formed. If this process does not involve artificial tamping the value of $K_o$ ranges from about 0.40 for loose sand to 0.6 for dense sand. Tamping the layers may increase it to 0.8.

The value of $K_o$ may also be obtained on the basis of elastic theory. If a cylindrical sample of soil is acted upon by vertical stress $\sigma_v$, and horizontal stress $\sigma_h$, the lateral strain $e_1$ may be expressed as

$$e_1 = \frac{1}{E} \left( \sigma_h - \mu (\sigma_h + \sigma_v) \right)$$

where $E$ = Young’s modulus, $\mu$ = Poisson’s ratio.

The lateral strain $e_1 = 0$ when the earth is in the at rest condition. For this condition, we may write

$$\frac{1}{E} \left( \sigma_h - \mu (\sigma_h + \sigma_v) \right) = 0 \quad \text{or} \quad \frac{\sigma_h}{\sigma_v} = \frac{\mu}{1-\mu}$$

or

$$\sigma_h = \left( \frac{\mu}{1-\mu} \right) \sigma_v = K_0 \sigma_v = K_0 \gamma z$$

where

$$\frac{\mu}{1-\mu} = K_0, \quad \sigma_v = \gamma z$$

According to Jaky (1944), a good approximation for $K_0$ is given by Eq. $K_0 = 1 - \sin \phi$

Which fits the most experimental data

Sherif, Fang, and Sherif (1984),

$$K_o = (1 - \sin \phi) + \left[ \frac{\gamma_d}{\gamma_d(\min)} - 1 \right] 5.5$$

where $\gamma_d$ = actual compacted dry unit weight of the sand behind the wall

$\gamma_d(\min)$ = dry unit weight of the sand in the loosest state

Mayne and Kulhawy (1982)

$$K_o = (1 - \sin \phi')(OCR)^{\sin \phi'}$$

For fine-grained, normally consolidated soils, Massarsch (1979)

$$K_o = 0.44 + 0.42 \left[ \frac{PI (\%)}{100} \right]$$

Other Coorrelations are:

$K_o = (1 - \sin \phi')(Jaky, 1944)$

$K_o = 0.9 (1 - \sin \phi')(Fraser, 1957)$

$K_o = 0.19 + 0.233 \log I_p (Kenney, 1959)$

$K_o = [1 + (2/3) \sin \phi'] \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) (Kezdi, 1962)$

$K_o = (0.95 - \sin \phi')(Brooker and Ireland, 1965)$

For reference Only (Make your own notes)
3.4. Classical Earth Pressure Theories

Rankine’s Theory

Assumptions made:
1. The backfill soil is isotropic, homogeneous and is cohesionless.
2. The soil is in a state of plastic equilibrium during active and passive earth pressure conditions.
3. The rupture surface is a planar surface which is obtained by considering the plastic equilibrium of the soil.
4. The backfill surface is horizontal.
5. The back of the wall is vertical.
6. The back of the wall is smooth.

(a) Active state
Let XY in above Fig. (a) Active state, represent the horizontal surface of a semi-infinite mass of cohesionless soil with a unit weight $g$. The soil is in an initial state of elastic equilibrium. Consider a prismatic block ABCD. The depth of the block is $z$ and the cross-sectional area of the block is unity. Since the element is symmetrical with respect to a vertical plane, the normal stress on the base AD is $\sigma_{v} = \tau z$.

If the wall yields away from the soil, then the soil in the back expands thereby decreasing the horizontal stress. If the yield is large enough, the lateral stress decreases to a minimum value in which Mohr’s circle drawn touches the failure envelope. This is known as state of plastic equilibrium and the soil mass is said to be in active Rankine state.

(b) Passive state
For reference Only (Make your own notes)
If we imagine that the entire mass is subjected to horizontal deformation, such deformation is a plane deformation. Every vertical section through the mass represents a plane of symmetry for the entire mass. Therefore, the shear stresses on vertical and horizontal sides of the prism are equal to zero.

Due to the stretching, the pressure on vertical sides $AB$ and $CD$ of the prism decreases until the conditions of plastic equilibrium are satisfied, while the pressure on the base $AD$ remains unchanged. Any further stretching merely causes a plastic flow without changing the state of stress. The transition from the state of plastic equilibrium to the state of plastic flow represents the failure of the mass. Since the weight of the mass assists in producing an expansion in a horizontal direction, the subsequent failure is called active failure.

If, on the other hand, the mass of soil is compressed, as shown in Fig. (b) Passive state, in a horizontal direction, the pressure on vertical sides $AB$ and $CD$ of the prism increases while the pressure on its base remains unchanged at $yz$. Since the lateral compression of the soil is resisted by the weight of the soil, the subsequent failure by plastic flow is called a passive failure.

**Rankine’s Theory of Active Pressure**

The stress condition in the soil element can be represented by the Mohr’s circle in an adjacent figure. However, if the wall is allowed to move away from the soil mass gradually, the horizontal principal stress will decrease.

Ultimately a state will be reached when the stress condition in the soil element can be represented by the Mohr’s circle $b$, the state of plastic equilibrium and failure of the soil will occur. This situation represents Rankine’s active state, and the effective pressure on the vertical plane is Rankine’s active earth pressure. We can derive in terms of $\gamma$, $z$, $c$ (cohesion), and $\phi$ from the figure.

$$\sin \phi' = \frac{CD}{AC} = \frac{CD}{AO + OC}$$

$CD$ = radius of the failure circle $= \frac{\sigma_o' - \sigma_a'}{2}$

From figure, $AO = \frac{\sigma_o'}{\cos \phi'}$

$$\frac{\sigma_o'}{2} - \frac{\sigma_o' + \sigma_a'}{2}$$

$$\sin \phi' = \frac{\frac{\sigma_o'}{2} - \frac{\sigma_o' + \sigma_a'}{2}}{c' \cot \phi' + \frac{\sigma_o' + \sigma_a'}{2}}$$

$$c' \cos \phi' + \frac{\sigma_o' + \sigma_a'}{2} \sin \phi' = \frac{\sigma_o' - \sigma_a'}{2}$$

$$\sigma_o' = \sigma_o' \frac{1 - \sin \phi'}{1 + \sin \phi'} - 2c' \frac{\cos \phi'}{1 + \sin \phi'}$$

$\sigma_o' = \gamma z$ vertical effective overburden pressure

Therefore, $\tan^2 \left( \frac{45 - \phi'}{2} \right)$

In the case when $c'=0$, the relationship will be

$$\sigma_o' = \frac{\gamma z}{2} \tan^2 \left( \frac{45 - \phi'}{2} \right)$$

For reference Only (Make your own notes)
Rankine’s Theory of Passive Pressure (Do yourself)

\[
\sigma_p' = \sigma'_n \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)
\]

Active case for cohesionless soil with sloping backfill surface

Figure a, b and c, below shows a smooth vertical wall with a sloping backfill of cohesionless soil. As in the case of a horizontal backfill, the active state of plastic equilibrium can be developed in the backfill by rotating the wall about A away from the backfill. Let AC be the rupture line and the soil within the wedge ABC be in an active state of plastic equilibrium.

Consider a rhombic element E within the plastic zone ABC which is shown to a larger scale outside. The base of the element is parallel to the backfill surface which is inclined at an angle \( \beta \) to the horizontal. The horizontal width of the element is taken as unity.

\[
\sigma_r = \frac{\gamma H}{1/ \cos \beta} = \gamma \frac{H}{\cos \beta}
\]

Let \( \sigma_v \) = the vertical stress acting on an elemental length \( ab = \gamma \frac{H}{\cos \beta} \)

\( \sigma_r \) = the lateral pressure acting on vertical surface \( bc \) of the element

The vertical stress \( \sigma_v \) can be resolved into components \( \sigma_z \) the normal stress and \( \tau \) the shear stress on surface \( ab \) of element E. We may now write

\[
\sigma_n = \sigma_v \cos \beta = \gamma \frac{H}{\cos \beta} \cos \beta = \gamma \frac{H}{\cos \beta} \cos \beta
\]

\[
\tau = \sigma_v \sin \beta = \gamma \frac{H}{\cos \beta} \sin \beta
\]

A Mohr diagram can be drawn as shown in Fig. (c). Here, length \( OA = \gamma \frac{H}{\cos \beta} \) makes

For reference Only (Make your own notes)
an angle $\beta$ with the $\sigma$-axis. $OD = \sigma_y = \gamma \cos^2 \beta$ and $AD = \tau = \gamma \cos \beta \sin \beta$. $OM$ is the Mohr envelope making an angle $\phi$ with the $\sigma$-axis. Now Mohr circle $C_1$ can be drawn passing through point $A$ and at the same time tangential to envelope $OM$. This circle cuts line $OA$ at point $B$ and the $\sigma$-axis at $E$ and $F$.

The following relationships can be expressed with reference to the Mohr diagram:

$$BC = CA = \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$
$$\sigma_1 = OA = OC + CA = \frac{\sigma_1 + \sigma_3}{2} \cos \beta + \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$
$$\sigma_2 = p_a = OC - BC = -\frac{\sigma_1 + \sigma_3}{2} \cos \beta - \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta}$$

Now we have (after simplification)

$$\frac{\sigma_1}{\sigma_2} = \frac{p_a}{\gamma z \cos \beta} = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$

or

$$p_a = \gamma z \cos \beta \times \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} = \gamma z K_A$$

where, $K_A = \cos \beta \times \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$ is called as the coefficient of earth pressure for the active state or the active earth pressure coefficient.

Sometimes it is also referred as conjugate ratio.

**Sloping Surface- Passive Earth Pressure (Do yourself)**

An equation for $p_p$ for a sloping backfill surface can be developed in the same way as for an active case. The equation for $p_p$ may be expressed as

$$p_p = \frac{1}{2} \gamma H^2 K_p$$

where, $K_p = \cos \beta \times \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$

$p_p$ acts at a height $H/3$ above point $A$ and parallel to the sloping surface.
**COULOMB'S EARTH PRESSURE THEORY**

More than 200 years ago, Coulomb (1776) presented a theory for active and passive earth pressures against retaining walls. In this theory, Coulomb assumed that the failure surface is a plane. The *wall friction* was taken into consideration. The following sections discuss the general principles of the derivation of Coulomb’s earth-pressure theory for a cohesionless backfill (shear strength defined by the equation \( \tau_f = \sigma' \tan \phi' \)).

Thus the assumptions made by Coulomb can be summarized as:
1. The soil is isotropic and homogeneous
2. The rupture surface is a plane surface
3. The failure wedge is a rigid body
4. The pressure surface is a plane surface
5. There is wall friction on the pressure surface
6. Failure is two-dimensional and
7. The soil is cohesionless and the backfill surface can be inclined.

Let \( AB \) (Figure 13.22a) be the back face of a retaining wall supporting a granular soil; the surface of which is constantly sloping at an angle \( \alpha \) with the horizontal. \( BC \) is a trial failure surface. In the stability consideration of the probable failure wedge \( ABC \), the following forces are involved (per unit length of the wall):

1. \( W \)—the weight of the soil wedge.
2. \( F \)—the resultant of the shear and normal forces on the surface of failure, \( BC \). This is inclined at an angle of \( \phi' \) to the normal drawn to the plane \( BC \).
3. \( P_\alpha \)—the active force per unit length of the wall. The direction of \( P_\alpha \) is inclined at an angle \( \delta' \) to the normal drawn to the face of the wall that supports the soil. \( \delta' \) is the angle of friction between the soil and the wall.

![Figure 13.22 Coulomb's active pressure: (a) trial failure wedge; (b) force polygon](image)

For reference Only (Make your own notes)
The force triangle for the wedge is shown in Figure 13.22b. From the law of sines, we have
\[
\frac{W}{\sin(90 + \theta + \delta' - \beta + \phi')} = \frac{P_a}{\sin(\beta - \phi')}
\]
or
\[
P_a = \frac{\sin(\beta - \phi')}{\sin(90 + \theta + \delta' - \beta + \phi')} W
\]
The preceding equation can be written in the form
\[
P_a = \frac{1}{2} \gamma H^2 \left[ \frac{\cos(\theta - \beta)\cos(\theta - \alpha)\sin(\beta - \phi')}{\cos^2 \theta \sin(\beta - \alpha)\sin(90 + \theta + \delta' - \beta + \phi')} \right]
\]
where \( \gamma \) = unit weight of the backfill. The values of \( \gamma, H, \theta, \alpha, \phi', \) and \( \delta' \) are constants, and \( \beta \) is the only variable. To determine the critical value of \( \beta \) for maximum \( P_a \), we have
\[
\frac{dP_a}{d\beta} = 0
\]
After solving
\[
P_a = \frac{1}{2} K_a \gamma H^2
\]
where \( K_a \) is Coulomb’s active earth-pressure coefficient and is given by
\[
K_a = \frac{\cos^2(\phi' - \theta)}{\cos^2 \theta \cos(\delta' + \theta) \left[ 1 + \sqrt{\frac{\sin(\delta' + \phi')\sin(\phi' - \alpha)}{\cos(\delta' + \theta)\cos(\theta - \alpha)}} \right]^2}
\]
Similarly, the Coulomb’s passive earth pressure coefficient is given by
\[
K_p = \frac{\cos^2(\phi' + \theta)}{\cos^2 \theta \cos(\delta' - \theta) \left[ 1 - \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' + \alpha)}{\cos(\delta' - \theta)\cos(\alpha - \theta)}} \right]^2}
\]

3.5 Yielding of Wall of Limited Height

We learned in the preceding discussion that sufficient movement of a frictionless wall extending to an infinite depth is necessary to achieve a state of plastic equilibrium. However, the distribution of lateral pressure against a wall of limited height is influenced very much by the manner in which the wall actually yields. In most retaining walls of limited height, movement may occur by simple translation or, more frequently, by rotation about the bottom.
3.6 Graphical solution for coulomb’s earth pressure

ACTIVE PRESSURE BY CULMANN’S METHOD FOR COHESIONLESS SOILS

Culmann’s (1875) method is the same as the trial wedge method. In Culmann's method, the force polygons are constructed directly on the $\phi$-line AE taking AE as the load line. The procedure is as follows:

1. Draw $\phi$ -line $AE$ at an angle $\phi$ to the horizontal.
2. Lay off on $AE$ distances, $AV$, $A1$, $A2$, $A3$, etc. to a suitable scale to represent the weights of wedges $ABV$, $AB1$, $AB2$, $AB3$, etc. respectively.
3. Draw lines parallel to $AD$ from points $V$, $1$, $2$, $3$ to intersect assumed rupture lines $AV$, $A1$, $A2$, $A3$ at points $V'$, $1'$, $2'$, $3'$, etc. respectively.
4. Join points $V'$, $1'$, $2'$, $3'$, etc. by a smooth curve which is the pressure locus.
5. Select point $C'$ on the pressure locus such that the tangent to the curve at this point is parallel to the $\phi$-line $AE$.
6. Draw $C'\overline{C}$ parallel to the pressure line $AD$. The magnitude of $C'\overline{C}$ in its natural units gives the active pressure $P_\alpha$.
7. Join $AC'$ and produce to meet the surface of the backfill at $C$. $AC$ is the rupture line.
For Passive case (Practice yourself)

- Draw $\phi$-line AE at an angle $\phi$ below the horizontal.
- Lay off on AE distances $A_2$, $A_3$, $A_4$ etc to a suitable scale to represent the weight of wedges $AB_2$, $AB_3$, $AB_4$, and so on.
- Lay off AD at an angle equal to ($\alpha+\delta$) to the line AE. The line AD is called pressure line.
- Draw lines parallel to AD from points 2, 3, 4 etc to intersect the weight vectors $A_2$, $A_3$, $A_4$ at points $2'$, $3'$, $4'$ etc respectively.
- Join points $2'$, $3'$, $4'$ etc by a smooth curve which is the pressure locus.
- Select the point C' on pressure locus curve such that the line tangent to the curve is parallel to $\phi$-line AE.
- Draw CC' parallel to the pressure line AD. The magnitude of CC' in its natural units gives the passive pressure $P_p$.

Join AC'. The line cuts the surface of the backfill at C. The line AC is the rupture line.

Other graphical Methods are

Rehhann’s Method
3.7 Trial wedge method for earth pressure

As previously noted, the trial wedge and Culmann procedures are identical except for orientation of the force polygon. The trial wedge also has an advantage over the Culmann solution since one can have cohesion as a soil parameter. Figure 11-14 illustrates the general procedure, which may be outlined as follows:

1. Draw the wall and ground surface to a scale that is as large as possible and compute the depth of the tension crack as

\[ h_t = \frac{2c}{\gamma \sqrt{K_a}} \]

This value of \( h_t \) is then plotted at sufficient points to establish the tension-crack profile (dashed line \( BD_1D_2D \) of Fig. 11-14a).

2. Lay off trial wedges as \( AB'EC_1D_1, AB'EC_2D_2, \ldots \), and compute the weight of the corresponding wedges as \( w_1, w_2, \ldots, w_n \). With a tension crack it may be preferable to compute the weights as the sum of the tension block plus the weight of the triangle (as in Ex. 11-6).

3. Compute \( C_w \) and \( C_s \) (note that \( C_w \) is a constant) and lay off \( C_w \) as indicated in Fig. 11-14b to the wall slope and to the appropriate force scale. As a tension crack can form along the wall, the length \( AB \) (and not \( AB' \)) should be used to compute \( C_w \). Also draw the weight vectors \( w_1, w_2, \ldots, w_n \) along the line \( OY \). Note that the slopes are transferred from the wedge to the force polygon.

4. From the terminus of \( C_w \) lay off \( C_s \) at the slope of the assumed trial failure wedges.

5. Through points \( w_1, w_2, \ldots, w_n \) established in step 3, lay off a vector \( P_a \) to the correct slope. Note that the slope of \( P_a \) (or \( P_p \)) is constant.

6. Through the terminus of \( C_s \) lay off the vector \( R \) to the appropriate slope. The slope is at the angle \( \phi \) to a perpendicular to the assumed failure surfaces \( AD_1, AD_2, AD_3, \ldots \).

7. The intersection of the \( R \) and \( P_a \) vectors establishes a locus of points, through which a smooth curve is drawn.

8. Draw a tangent to the curve obtained in step 7, parallel to the weight vector, and draw the vector \( P_a \) through the point of tangency. As with the Culmann solution, several maximum values may be obtained. The largest possible value of \( P_a \) is the design value.

The slope of the \( R \) vector (step 6 preceding) can be established conveniently (Fig. 11-14c) as follows:

1. To some radius \( r \) draw the arc \( GJ \) from the vertical line \( AF \) in Fig. 11-14a.

2. Draw a horizontal line \( AO \) and lay off the angle \( \phi \) as shown. With the same \( r \) used in step 1, draw arc \( OJ \) using \( A \) as the center.

3. Then \( AG \) is the slope of the vector \( R \) to failure plane \( AF \).

4. Now lay off arcs \( GH, HI, IJ \) in Fig. 11-14c to the same arc length used in step 1.

5. The slopes of lines \( AH, AI, \) and \( AJ \) of Fig. 11-14c are the corresponding slopes of the vector \( R \) to failure surface \( AD_1, AD_2, \ldots \).

In cohesionless materials the values \( C_w \) and \( C_s \) are zero, and the trial wedge solution is the same as the Culmann solution except for the orientation of the force polygon.
Example of Trial Wedge Method

Calculate $n_r = \frac{30}{19.3} = 1.58$ m

Calculate cohesion $C_r$

$C_{r1} = 4.6(0.10) = 46$ kN
$C_{r2} = 5.7(0.10) = 57$ kN
$C_{r3} = 6.0(0.10) = 60$ kN
$C_{r4} = 6.4(0.10) = 64$ kN
$C_{r5} = 7.0(0.10) = 70$ kN
$C_{r6} = 7.6(0.10) = 76$ kN
$C_{r7} = 8.3(0.10) = 84$ kN
$C_{r8} = 9.1(0.10) = 91$ kN

$W_1 = \text{Triangle crack break} + \text{Triangle } 1 + \Sigma W_{1}$

$W_1 = 1.2 \times 34.2 \times 19 + \frac{1}{4} (5.7 \times 0.97)(19) = 86.7 \times 0.0 \times 168.3$

$W_2 = 1 \times 28.5 \times 19 + \frac{1}{4} (6.0 \times 0.93)(19) = 81.5 \times 86.7 \times 168.2$

$W_{10} = 30.0 = 30.0 \times 168.2$

$W_{12} = 28.5 + \frac{1}{2} (7.0 \times 0.79)(19) = 82.0 \times 446.2$

$W_{20} = 40 = 528.2$

$W_2 = 1 \times 28.5 \times 19 + \frac{1}{2} (9.1 \times 0.61)(19) = 81.2 \times 568.2 \times 649.4$

Figure E11-6

(a) Forces acting on a trial wedge $ABED$

(b) Forces acting on $AB'ED$ formed into the force polygon

(c) Rapid method of establishing the slope of $R$

$C_w = AB \cdot \text{cohesion (direction and magnitude known)}$

$C_r = AD \cdot \text{cohesion (direction and magnitude known)}$

$W = \text{weight of trial wedge (direction and magnitude known)}$

$R = \text{known in direction}$

$P_r = \text{known in direction}$

Figure 11-14 The trial wedge active force solution. For passive force slope of $P_r$ is shown; slope $R$ changes, $C_r, C_w$ reverse directions.

For reference Only (Make your own notes) 13
3.8. Proportioning of retaining walls
Based on practical experience, retaining walls can be proportioned initially which may be checked for stability subsequently.

- Proportioning is an assumption which allow the engineer to check trial sections of the walls for stability.
- If the stability checks yield undesirable results, the sections can be changed and rechecked.
- The top of the stem of any retaining wall should not be less than about 0.3m for proper placement of concrete.
- The top of the stem of any retaining wall should not be less than about 0.3m for proper placement of concrete.
- The depth to the bottom of the base slab should be a minimum of 0.6m
- The bottom of the base slab should be positioned below the seasonal frost line.
- The counterfort slabs may be 0.3m thick and spaced at center to center distances of 0.3H to 0.7H and width of 0.4 to 0.7H.

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<th>Banded Masonry</th>
<th>Cement Masonry</th>
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<th>Reinforced Earth</th>
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<td>0.6–1.0</td>
<td>0.5–1.0</td>
<td>1.0</td>
<td>4.0 or 0.7–0.8H</td>
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<td>0.5–0.7H</td>
<td>0.6–0.65H</td>
<td>0.5–0.65H</td>
<td>0.6–0.75H</td>
<td>4.0 or 0.7–0.8H</td>
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<td>Front batter (V:H)</td>
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<tr>
<td>Back batter (V:H)</td>
<td>4:1</td>
<td>vertical</td>
<td>vertical</td>
<td>varies</td>
<td>varies</td>
<td></td>
</tr>
<tr>
<td>Foundation dip (V:H)</td>
<td>1:4</td>
<td>1:3</td>
<td>1:3</td>
<td>1:10–1:6</td>
<td>1:6–1:4</td>
<td>horizontal</td>
</tr>
<tr>
<td>Foundation depth (m)</td>
<td>0.5–1.0</td>
<td>0.5</td>
<td>0.5–1.0</td>
<td>0.5–1.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Height range (m)</td>
<td>4.0–12.0</td>
<td>1.0–4.0</td>
<td>4.0–8.0</td>
<td>1.0–10.0</td>
<td>1.0–6.0</td>
<td>3.0–12.0</td>
</tr>
<tr>
<td>Fill slope angle (°)</td>
<td>&lt; 30°</td>
<td>&lt; 30°</td>
<td>&lt; 20°</td>
<td>35°–60°</td>
<td>35°–60°</td>
<td>&lt; 35°</td>
</tr>
</tbody>
</table>

Source: Adapted from MREH
For reference Only (Make your own notes)
3.9 Stability of retaining walls

The stability of retaining walls should be checked for the following conditions:

- The assumption for the development of Rankines active earth pressure along the soil face AB is theoretically correct if the shear zone bounded by the line AC is not obstructed by the stem of the wall.
- A similar type of analysis may be used for gravity walls, however Coulomb’s earth pressure theory also may be used.

$$\eta = 45 + \frac{\alpha}{2} - \frac{\phi'}{2} - \frac{1}{2} \sin^{-1} \left( \frac{\sin \alpha}{\sin \phi'} \right)$$

- While using Rankine theory for the Wall’s stability, the active and passive forces, the weight of the soil above the heel and the concrete weight, all should be taken into consideration.
- If Coulomb’s theory is used, the only forces to be considered are active and passive forces and the weight of the wall.
- If Coulomb’s theory is used, it will be necessary to know the range of the wall friction angle with various types of backfill material.

<table>
<thead>
<tr>
<th>Backfill material</th>
<th>Range of $\delta'$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>27–30</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>20–28</td>
</tr>
<tr>
<td>Fine sand</td>
<td>15–25</td>
</tr>
<tr>
<td>Stiff clay</td>
<td>15–20</td>
</tr>
<tr>
<td>Silty clay</td>
<td>12–16</td>
</tr>
</tbody>
</table>

Stability of retaining wall must be checked a) by overturning, b) by sliding, c) by bearing capacity failure and d) by deep seated failure.

1. **Check for overturning**

Due to the lateral force, there is possibility of overturning of retaining wall w.r.t. the toe of the wall. So the force causing the overturning of wall is the moment due to the horizontal force which is anticlockwise in nature and the force resisting this action is the clockwise moment.

*For reference Only (Make your own notes)*
taking toe as the point of rotation. Mathematically,
The factor of safety against overturning about point (Toe)
may be expressed as

\[ FS_{\text{overturning}} = \frac{\Sigma M_R}{\Sigma M_I} \]

Where,
\( \Sigma M_O = \text{sum of the moments of forces tending to overturn about point C (Toe)} \)
\( \Sigma M_R = \text{sum of the moments of forces tending to resist overturning about point C (Toe)} \)

\[ \Sigma M_o = P_a \left( \frac{H'}{3} \right) \quad \text{where} \quad P_a = P \cos \alpha. \]

\[ FS_{\text{overturning}} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7}{P_a \cos \alpha \left( \frac{H'}{3} \right)} \]

NOTE: The usual minimum desirable value of the factor of safety with respect to overturning is 2 to 3.

2. Check for sliding
The force causing sliding is the horizontal component of lateral earth pressure acting on the
back of the wall and the forces resisting the sliding action are the passive force generated in the
front (which is generally neglected for the safe design), the friction on the base of the wall and the
adhesion on the base. Mathematically

\[ FS_{\text{sliding}} = \frac{\Sigma F_R}{\Sigma F_d} \]

where
\( \Sigma F_R = \text{sum of the horizontal resisting forces} \)
\( \Sigma F_d = \text{sum of the horizontal driving forces} \)

The shear strength of the soil immediately below the base slab may be represented as

\[ s = \sigma' \tan \delta' + c'_d \]

where
\( \delta' = \text{angle of friction between the soil and the base slab} \)
\( c'_d = \text{adhesion between the soil and the base slab} \)

The maximum resisting force that can be derived from the soil per unit length of the wall along the
bottom of the base slab is

\[ R' = s (\text{area of cross section}) = s (B \times 1) = B \sigma' \tan \delta' + B c'_d \]
\[ B \sigma' = \text{sum of the vertical force} = \Sigma V \]
\[ R' = (\Sigma V) \tan \delta' + B c'_d \]

In the above figure the passive pressure \( P_p \) is also horizontal resisting force

\[ \Sigma F_R = (\Sigma V) \tan \delta' + B c'_d + P_p \]

The horizontal force that will tend to cause the wall to slide (a driving force) is the horizontal
component of the active force \( P_a \)

\[ \Sigma F_d = P_a \cos \alpha \quad FS_{\text{sliding}} = \frac{(\Sigma V) \tan \delta' + B c'_d + P_p}{P_a \cos \alpha} \]
Note: Factor of safety against sliding is generally taken as 1.5.

3. Check for bearing capacity failure
The stress induced due to the loading including weight of retaining wall itself at the base level soil shouldn’t be greater than the allowable bearing capacity of the soil. For simplicity, the variation of soil pressure is assumed to vary linearly.

\[ R = \Sigma V + P_h \]

- The net moment of these forces about point C

\[ M_{net} = \Sigma M_R - \Sigma M_n \]

\[ CE = \frac{x}{\Sigma V} = \frac{M_{net}}{\Sigma V} \]

\[ e = \frac{B}{2} - CE \]

- The pressure distribution under the base slab may be determined by using principles from the mechanics of materials

\[ q = \frac{\Sigma V}{A} + \frac{M_{maxy}}{l} \]

\[ M_{net} = \text{moment} = (\Sigma V)e \]

\[ l = \text{moment of inertia per unit length of the base section} = \frac{1}{12}(B') \]

\[ q_{max} = q_{maxx} = \frac{\Sigma V}{(B)(1)} + \frac{e(\Sigma V)}{(B')} = \frac{\Sigma V}{B}(1 + \frac{6e}{B}) \]

\[ q_{min} = q_{bol} = \frac{\Sigma V}{B}(1 - \frac{6e}{B}) \]

The above formulae only applies when \( \Sigma V \) or \( Rv \) is within the middle third

\[ \frac{Rv}{B}(1 + \frac{6e}{B}) \]

\[ \frac{Rv}{B}(1 - \frac{6e}{B}) \]

\[ \frac{2Rv}{3B} \]

(a) Within middle third  
(b) On middle third  
(c) Outside middle third

When \( R_v \) or \( \Sigma V \) lies on middle third then, \( e = \frac{B}{6} \)

Maximum pressure = \( \frac{2R_v}{B} \); Minimum pressure = 0

When \( R_v \) or \( \Sigma V \) lies outside the middle third then, \( e = \frac{B}{2} - \frac{e}{6} \)

Maximum pressure = \( \frac{2R_v}{3B} \); Minimum pressure = 0

and the maximum soil reaction \( (q_{s-max}) \) is given

\[ q_{s-max} = \frac{2V}{3B(\frac{B}{2} - e)} \]

\[ FOS \text{ against bearing capacity} = \frac{q_{na}}{q_{max}} \]

For reference Only (Make your own notes)
NOTE: Factor of safety against bearing capacity failure ≥ 3.0

4. Check for tension
   If the eccentricity is greater than B/6, where B is the width of the retaining wall, then the minimum soil pressure is negative that mean tension. But we know that soil cannot bear any tension, the contact area between wall base and soil decreases thus leading to failure of wall system.
   Hence for safety, Eccentricity (e) ≤ B/6